REQUIRED FORMULAE

Roots of polynomials		
Formula	**Comment**	**Papers**
$ax^2 + bx + c = 0$ has roots $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$		2, 3
For $ax^2 + bx + c = 0$ with roots α and β: $\alpha + \beta = -b/a, \;\; \alpha\beta = c/a$		2, 3
For $ax^3 + bx^2 + cx + d = 0$ with roots α, β and γ: $\alpha + \beta + \gamma = -b/a, \;\; \alpha\beta + \beta\gamma + \gamma\alpha = c/a, \;\; \alpha\beta\gamma = -d/a$	The pattern is the same for polynomial equations of higher degree	2, 3

Laws of indices		
Formula	**Comment**	**Papers**
$a^x a^y = a^{x+y}$		2, 3
$a^0 = 1$	$a \neq 0$	2, 3
$(a^x)^y = a^{xy}$		2, 3
$a^x = \mathrm{e}^{x \ln a}$	defines a^x when x is not an integer	2, 3 •

Laws of logarithms		
Formula	**Comment**	**Papers**
$x = a^n \Leftrightarrow n = \log_a x$	$x > 0$, $a > 0$ $(a \neq 1)$	2, 3
$\log_a x + \log_a y = \log_a(xy)$		2, 3
$\log_a x - \log_a y = \log_a(x/y)$		2, 3
$k \log_a x = \log_a x^k$	for $x > 0$	2, 3

Sequences and series		
Formula	**Comment**	**Papers**
General (nth) term of an arithmetic progression: $u_n = a + (n-1)d$	d is the common difference	2, 3
General (nth) term of a geometric progression: $u_n = ar^{n-1}$	r is the common ratio	2, 3
Sum of an arithmetic progression: $S_n = \frac{1}{2}n\{2a + (n-1)d\}$	or: $S_n = an + \frac{1}{2}n(n-1)d$	2, 3 •
Sum of a geometric progression: $S_n = \dfrac{a(1-r^n)}{1-r}$		2, 3 •
Sum to infinity of a geometric progression: $S_\infty = \dfrac{a}{1-r}$	$\lvert r\rvert < 1$	2, 3 •
${}^nC_r = \dfrac{n!}{(n-r)!r!}$		2, 3 •
$(a+b)^n = \displaystyle\sum_{r=0}^{n} {}^nC_r a^{n-r} b^r$	Binomial expansion, $n \in \mathbb{N}$	2, 3 •
$(1+x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 + \cdots + \frac{k(k-1)\cdots(k-r+1)}{r!}x^r + \cdots$	$\lvert x\rvert < 1$, $k \in \mathbb{Q}$	2, 3 •
$\displaystyle\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$		2, 3 •
$\mathrm{f}(x) = \displaystyle\sum_{r=0}^{\infty} \frac{1}{r!}\,\mathrm{f}^{(r)}(0)\,x^r$	Maclaurin series	3 •
$\mathrm{e}^x = \displaystyle\sum_{r=0}^{\infty} \frac{x^r}{r!}$	converges for all x	2, 3 •
$\ln(1+x) = \displaystyle\sum_{r=1}^{\infty} (-1)^{r+1}\frac{x^r}{r}$	converges for $-1 < x \leqslant 1$	3 •
$\sin x = \displaystyle\sum_{r=0}^{\infty} (-1)^r \frac{x^{2r+1}}{(2r+1)!}$	converges for all x	3 •
$\cos x = \displaystyle\sum_{r=0}^{\infty} (-1)^r \frac{x^{2r}}{(2r)!}$	converges for all x	3 •

Coordinate geometry		
Formula	**Comment**	**Papers**
The straight line graph with gradient m passing through the point (x_1, y_1) has equation $y - y_1 = m(x - x_1)$		2, 3
Straight lines with non-zero gradients m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$		2, 3

Trigonometry		
Formula	**Comment**	**Papers**
Sine rule for the triangle ABC: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$		2, 3
Cosine rule in the triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$		2, 3
Area of triangle ABC: $\frac{1}{2}ab\sin C$		2, 3
$\cos^2 A + \sin^2 A = 1$		2, 3
$\sec^2 A = 1 + \tan^2 A$		2, 3
$\text{cosec}^2 A = 1 + \cot^2 A$		2, 3
$\sin 2A = 2\sin A\cos A$		2, 3
$\cos 2A = \cos^2 A - \sin^2 A$		2, 3
$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$	$A \neq (k + \frac{1}{2})\frac{\pi}{2}$, $k \in \mathbb{Z}$	2, 3
$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$		2, 3
$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$		2, 3
$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$	$A \pm B \neq (k + \frac{1}{2})\pi$, $k \in \mathbb{Z}$	2, 3 •
$\sin\theta \approx \theta\,, \quad \cos\theta \approx 1 - \frac{1}{2}\theta^2\,, \quad \tan\theta \approx \theta$	θ small (compared with 1); θ in radians	2, 3 •

Hyperbolic functions		
Formula	**Comment**	**Papers**
$\sinh x = \dfrac{e^x - e^{-x}}{2}$	by definition	3
$\cosh x = \dfrac{e^x + e^{-x}}{2}$	by definition	3
$\tanh x = \dfrac{\sinh x}{\cosh x}$	by definition	3
$\cosh^2 A - \sinh^2 A = 1$		3 •
$\text{sech}^2 A = 1 - \tanh^2 A$		3 •
$\text{cosech}^2 A = \coth^2 A - 1$		3 •
$\sinh 2A = 2 \sinh A \cosh A$		3 •
$\cosh 2A = \cosh^2 A + \sinh^2 A$		3 •
$\tanh 2A = \dfrac{2 \tanh A}{1 + \tanh^2 A}$		3 •
$\sinh(A \pm B) = \sinh A \cosh B \pm \cosh A \sinh B$		3 •
$\cosh(A \pm B) = \cosh A \cosh B \pm \sinh A \sinh B$		3 •
$\tanh(A \pm B) = \dfrac{\tanh A \pm \tanh B}{1 \pm \tanh A \tanh B}$		3 •

Derivatives			
Function	**Derivative**	**Comment**	**Papers**
$\sin x$	$\cos x$		2, 3
$\cos x$	$-\sin x$		2, 3
$\tan x$	$\sec^2 x$		2, 3 •
$\cot x$	$-\text{cosec}^2 x$		2, 3 •
$\sec x$	$\sec x \tan x$		2, 3 •
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$		2, 3 •
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$		2, 3 •
$\tan^{-1} x$	$\frac{1}{1+x^2}$		2, 3 •
$\sinh x$	$\cosh x$		3
$\cosh x$	$\sinh x$		3
$\tanh x$	$\text{sech}^2 x$		3 •
$\coth x$	$-\text{cosech}^2 x$		3 •
$\text{sech}\, x$	$-\text{sech}\, x \tanh x$		3 •
$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$		3 •
$\tanh^{-1} x$	$\frac{1}{1-x^2}$		3 •
e^x	e^x		2, 3
$\ln x$	$\frac{1}{x}$		2, 3
$f(x) + g(x)$	$f'(x) + g'(x)$		2, 3
$f(x)g(x)$	$f'(x)g(x) + f(x)g'(x)$	product rule	2, 3
$\frac{f(x)}{g(x)}$	$\frac{f'(x)g(x) - f(x)g'(x)}{\big(g(x)\big)^2}$	quotient rule	2, 3 •
$f\big(g(x)\big)$	$f'\big(g(x)\big)g'(x)$	chain rule	2, 3

Integrals			
Function	**Integral**	**Comment**	**Papers**
x^n	$\frac{1}{n+1}\, x^{n+1} + c$	$n \neq -1$	2, 3
x^{-1}	$\ln\lvert x\rvert + c$		2, 3
$\cos x$	$\sin x + c$		2, 3
$\sin x$	$-\cos x + c$		2, 3
$\sinh x$	$\cosh x + c$		3
$\cosh x$	$\sinh x + c$		3
$\dfrac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x + c$	$-1 < x < 1$	2, 3 •
$\dfrac{1}{1+x^2}$	$\tan^{-1} x + c$		2, 3 •
e^x	$\mathrm{e}^x + c$		2, 3
$\mathrm{f}'(x) + \mathrm{g}'(x)$	$\mathrm{f}(x) + \mathrm{g}(x) + c$		2, 3
$\mathrm{f}'\big(\mathrm{g}(x)\big)\, \mathrm{g}'(x)$	$\mathrm{f}\big(\mathrm{g}(x)\big) + c$		2, 3
$\dfrac{\mathrm{f}'(x)}{\mathrm{f}(x)}$	$\ln\lvert \mathrm{f}(x)\rvert + c$		2, 3 •
$\big(\mathrm{f}(x)\big)^n\, \mathrm{f}'(x)$	$\frac{1}{n+1}\big(\mathrm{f}(x)\big)^{n+1} + c$	$n \neq -1$	2, 3 •
$u\dfrac{\mathrm{d}v}{\mathrm{d}x}$	$uv - \displaystyle\int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$	integration by parts	2, 3 •

General calculus		
Formula	**Comment**	**Papers**
$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$	first principles definition	2, 3 •
$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \bigg/ \frac{\mathrm{d}x}{\mathrm{d}t}$	for the parameterised curve $y = y(t),\ x = x(t)$	2, 3 •
Area under the curve $y = \mathrm{f}(x)$ and above the x-axis: $\int_a^b \mathrm{f}(x)\,\mathrm{d}x$		2, 3
Volume of revolution about the x-axis: $\pi \int_a^b \big(\mathrm{f}(x)\big)^2\,\mathrm{d}x$		3
$\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h(y_0 + y_n) + h(y_1 + y_2 + \cdots + y_{n-1})$	$h = \frac{b-a}{n}$, $y_r = y(a + rh)$, trapezium rule	2, 3 •
$\ddot{x} = -\omega^2 x \Rightarrow x = R\sin(\omega t + \alpha)$ or $x = R\cos(\omega t + \beta)$ or $x = A\cos\omega t + B\sin\omega t$	simple harmonic motion	3 •

Circles		
Formula	**Comment**	**Papers**
Length of an arc of a circle of radius r: $r\theta$	θ is angle subtended in radians	2, 3
Area of a sector of a circle of radius r: $\frac{1}{2}r^2\theta$	θ is angle subtended in radians	2, 3

Complex numbers		
Formula	**Comment**	**Papers**
$e^{i\theta} = \cos\theta + i\sin\theta$		3
$z = r(\cos\theta + i\sin\theta) \Rightarrow z^n = r^n(\cos n\theta + i\sin n\theta)$	de Moivre's theorem	3 •
$z^n = 1$ has roots $z = e^{2\pi k i/n}$ where $k = 0, 1, \ldots, (n-1)$	Roots of unity	3 •
Half line with end-point a: $\arg(z-a) = \theta$	θ is the angle between the line and a line parallel to the positive real axis	2, 3
Circle, centre a and radius r: $\lvert z-a \rvert = r$		2, 3

Vectors		
Formula	**Comment**	**Papers**
$\lvert x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \rvert = \sqrt{x^2+y^2+z^2}$		2, 3
$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\cos\theta$	scalar product, $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$	2, 3
$\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$	vector product	3 •
$\lvert\mathbf{a} \times \mathbf{b}\rvert = \lvert\mathbf{a}\rvert\,\lvert\mathbf{b}\rvert\lvert\sin\theta\rvert$	θ is the acute angle between the vectors	3 •
Equation of the line through the point with position vector $\mathbf{a}$ parallel to $\mathbf{b}$: $\mathbf{r} = \mathbf{a} + t\mathbf{b}$		2, 3
Equation of the plane containing the point with position vector $\mathbf{a}$ and with normal $\mathbf{n}$: $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$		3

Matrices		
Formula	**Comment**	**Papers**
For $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\det \mathbf{A} = ad - bc$		2, 3
For $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\mathbf{A}^{-1} = \dfrac{1}{\det \mathbf{A}} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$	$\det \mathbf{A} \neq 0$	2, 3
$\mathbf{AB}$ is equivalent to $\mathbf{B}$ then $\mathbf{A}$	for transformations represented by these matrices	2, 3
$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$	$\det \mathbf{AB} \neq 0$	2, 3
$\begin{pmatrix} 0 & \pm 1 \\ \pm 1 & 0 \end{pmatrix}$	reflection in the line $y = \pm x$	2, 3 •
$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$	rotation by θ about the z-axis; the direction of positive rotation is taken to be anticlockwise when looking towards the origin from the positive side of the axis of rotation	2, 3 •
$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$	reflection in the plane $z = 0$	2, 3 •

Mechanics		
Formula	**Comment**	**Papers**
mg	weight	2, 3
$F \leqslant \mu R$	frictional force related to normal reaction R	2, 3
$F = ma$	scalar version of Newton's second law; constant mass	2, 3
$\mathbf{F} = m\mathbf{a}$	vector version of Newton's second law; constant mass	2, 3
$\frac{1}{2}mv^2$	kinetic energy	2, 3 •
mgh	change in gravitational potential energy; h is vertical height	2, 3 •
mv	momentum	2, 3 •
$mv - mu$	impulse	2, 3 •
$T = \dfrac{\lambda x}{l} = kx$	Hooke's law	2, 3 •
$E = \dfrac{\lambda x^2}{2l} = \dfrac{1}{2}kx^2$	elastic potential energy	2, 3 •
$v = \dfrac{\mathrm{d}r}{\mathrm{d}t},\ \ a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = \dfrac{\mathrm{d}^2 r}{\mathrm{d}t^2}$	motion in a straight line (where acceleration, a, may not be constant)	2, 3 •
$v = u + at,\ \ s = ut + \frac{1}{2}at^2,\ \ s = \frac{1}{2}(u+v)t,\ \ v^2 - u^2 = 2as$	motion in a straight line with constant acceleration, a	2, 3 •
$\mathbf{v} = \dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}t},\ \mathbf{a} = \dfrac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$	motion in two (STEP 2) or three (STEP 3) dimensions where acceleration, **a**, may not be constant	2, 3 •
$\mathbf{v} = \mathbf{u} + \mathbf{a}t,\ \mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2,\ \mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t,\ \mathbf{v.v} - \mathbf{u.u} = 2\mathbf{a.s}$	motion in two or three dimensions with constant acceleration, **a**	2, 3 •
$v_1 - v_2 = -e(u_1 - u_2)$ or relative speed of separation $= e \times$ relative speed of approach	Newton's experimental law	2, 3 •
speed $= r\dot{\theta}$, radial acceleration $= \dfrac{v^2}{r} = r\dot{\theta}^2$ towards the centre, tangential acceleration $= r\ddot{\theta}$	motion in a circle of radius r	3 •

Probability/Statistics		
Formula	**Comment**	**Papers**
$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$	probability of the union of two events	2, 3 •
$\mathrm{P}(A \cap B) = \mathrm{P}(A \mid B)\,\mathrm{P}(B)$	probability of the intersection of two events	2, 3 •
$\mathrm{E}(aX + bY + c) = a\mathrm{E}(X) + b\mathrm{E}(Y) + c$	algebra of expectation	3 •
$\mathrm{Var}\,(aX + bY + c) = a^2\,\mathrm{Var}\,(X) + b^2\,\mathrm{Var}\,(Y)$	algebra of variances for independent variables	3 •
$\mu = \mathrm{E}(X) = \sum_i x_i \mathrm{P}(X = x_i)$	expectation of a discrete random variable X	2, 3 •
$\mu = \mathrm{E}(X) = \int x\mathrm{f}(x)\,\mathrm{d}x$	expectation of a continuous random variable X with p.d.f. f	2, 3 •
$\sigma^2 = \mathrm{Var}\,(X) = \sum_i (x_i - \mu)^2 \mathrm{P}(X = x_i)$ $= \sum_i x_i^2\,\mathrm{P}(X = x_i) - \mu^2$	variance of a discrete random variable X	2, 3 •
$\sigma^2 = \mathrm{Var}\,(X) = \int (x - \mu)^2 \mathrm{f}(x)\,\mathrm{d}x = \int x^2 \mathrm{f}(x)\,\mathrm{d}x - \mu^2$	variance of a continuous random variable X with p.d.f. f	2, 3 •
$\mathrm{F}(x) = \mathrm{P}(X \leqslant x) = \int_{-\infty}^{x} \mathrm{f}(x)\,\mathrm{d}x$	cumulative distribution function (c.d.f.)	2, 3 •

Random variables				
Distribution	$\mathrm{P}(X=x)$	$\mathrm{E}(X)$	$\mathrm{Var}(X)$	**Papers**
Binomial $\mathrm{B}(n,p)$	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$	2, 3 •
Uniform distribution over 1, 2, …, n	$\frac{1}{n}$	$\frac{1}{2}(n+1)$	$\frac{1}{12}(n^2-1)$ (included for completeness; memorisation not required)	2, 3 •
Poisson $\mathrm{Po}(\lambda)$	$\frac{\lambda^x \mathrm{e}^{-\lambda}}{x!}$	λ	λ	2, 3 •
Distribution	p.d.f.	$\mathrm{E}(X)$	$\mathrm{Var}(X)$	**Papers**
Uniform distribution over $[a,b]$	$\frac{1}{b-a}$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$ (included for completeness; memorisation not required)	2, 3 •
Normal $\mathrm{N}(\mu,\sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}}\,\mathrm{e}^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ (included for completeness; memorisation not required)	μ	σ^2	2, 3 •