STEP III, 2024, Q8 EC

Question 8

One of the least popular questions in the Pure Mathematics section, candidates did slightly less well here than on question 7. There were some excellent answers to this question, but also some answers that were lacking in clear explanation. There were sometimes issues with candidates not understanding the direction of implication required by the various question parts. The best solutions used the structure of the question to help find appropriate and efficient methods to solve the problem but there were also some inventive solutions using other techniques.

Part (i) was generally done well, though some candidates did not show sufficient working to justify the given answer fully.

Part (ii) was also generally done well, but some candidates did not take advantage of the work done in the previous part to show that the given equation represented a pair of straight lines. A small minority of candidates instead tried to show that if the equation represented a pair of straight lines then $k = 1$.

Parts (iii) and (iv) were found to be more difficult.

In Part (iii) the most successful candidates tended to follow the lead of the previous parts and factorised the equation in part (ii) to find the equations of two straight lines. A considerable number of candidates made a sign error while doing this: expanding to check a factorisation is correct is always a good idea. Those that factorised usually could see how to set up two quadratic equations in x and so find a condition of s. Some candidates set up a quartic equation in x but only a small number of these could complete an argument to show that $s < -0.75$, and these candidates often were confused on the direction of implication needed in this part.

The direction of implication required in part (iv) confused a lot of candidates, with some stating that they had already answered this in the previous part and others repeating a proof that four distinct points implies $s < -0.75$. Some other candidates recognised that there must be two distinct points of intersection of the curves and each line but did not realise that one of these points of intersection could be where both curves and both lines meet. A sketch was often a good idea to help clarify the geometry of the situation. A handful of candidates managed to consider the "if and only if" situation by considering where the two straight lines were tangential to $y = x^2$ answering both of the last two parts in one go.