STEP III, 2024, Q6 EC

Question 6

This was the least popular of the Pure Mathematics section, and by a large margin the least successful of the whole paper.

Those candidates who were successful in part (i)(a) usually tackled the question by re-writing the differential equation as $\frac{d(x-y)}{dt} = -2(x - y)$. There were also some candidates who rewrote the equation as $\frac{dx}{dt} + 2x = \frac{dy}{dt} = 2y$ and used integrating factors effectively to solve this, although some integrated erroneously to achieve $x + 2xt = y + 2yt$. Some candidates correctly concluded that $x = y$ but did not go on to say that this implied that $x_0 = y_0$. Most of the candidates gaining no credit for this question substituted $x = y = 0$ into their differential equation and then integrated that.

In part (i)(b) those candidates who attempted it generally understood what was required, but some did not appreciate that the situation in this case had different initial conditions to that in part (a). Some candidates used the given differential equations to find a second order differential equation in x or y, which was a valid if inefficient method.

Those attempting part (ii) generally performed in a similar way to part (i), either gaining most of the credit available or making the same mistakes they had made in the previous parts. There were some candidates who rather cleverly spotted that they could combine the last two differential equations to show that $y = z$, and then show that $x = z$ and in so doing answer both parts (ii)(a) and (b) together.