

## STEP III, 2024, Q6

- 6 (i) A particle moves in two-dimensional space. Its position is given by coordinates  $(x, y)$  which satisfy

$$\begin{aligned}\frac{dx}{dt} &= -x + 3y + u \\ \frac{dy}{dt} &= x + y + u\end{aligned}$$

where  $t$  is the time and  $u$  is a function of time. At time  $t = 0$ , the particle has position  $(x_0, y_0)$ .

- (a) By considering  $\frac{dx}{dt} - \frac{dy}{dt}$ , show that if the particle is at the origin  $(0, 0)$  at some time  $t > 0$ , then it is necessary that  $x_0 = y_0$ .
- (b) Given that  $x_0 = y_0$ , find a constant value of  $u$  that ensures that the particle is at the origin at a time  $t = T$ , where  $T > 0$ .
- (ii) A particle whose position in three-dimensional space is given by co-ordinates  $(x, y, z)$  moves with time  $t$  such that

$$\begin{aligned}\frac{dx}{dt} &= 4y - 5z + u \\ \frac{dy}{dt} &= x - 2z + u \\ \frac{dz}{dt} &= x - 2y + u\end{aligned}$$

where  $u$  is a function of time. At time  $t = 0$ , the particle has position  $(x_0, y_0, z_0)$ .

- (a) Show that, if the particle is at the origin  $(0, 0, 0)$  at some time  $t > 0$ , it is necessary that  $y_0$  is the mean of  $x_0$  and  $z_0$ .
- (b) Show further that, if the particle is at the origin  $(0, 0, 0)$  at some time  $t > 0$ , it is necessary that  $x_0 = y_0 = z_0$ .
- (c) Given that  $x_0 = y_0 = z_0$ , find a constant value of  $u$  that ensures that the particle is at the origin at a time  $t = T$ , where  $T > 0$ .



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