

STEP III, 2024, Q2 MS

2. (i) (a)

$$\sqrt{4x^2 - 8x + 64} \leq |x + 8|$$

$$4x^2 - 8x + 64 \leq (x + 8)^2 = x^2 + 16x + 64$$

Thus

$$3x^2 - 24x = 3x(x - 8) \leq 0$$

M1



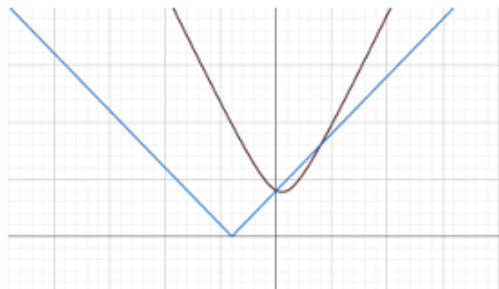
G1 or consideration of intervals M1

[Alternative method

Solve for critical values

M1

Sketch graph



G1]

So $0 \leq x \leq 8$

A1 (3)

(b)

$$\sqrt{4x^2 - 8x + 64} \leq |3x - 8|$$

$$4x^2 - 8x + 64 \leq (3x - 8)^2 = 9x^2 - 48x + 64$$

Thus

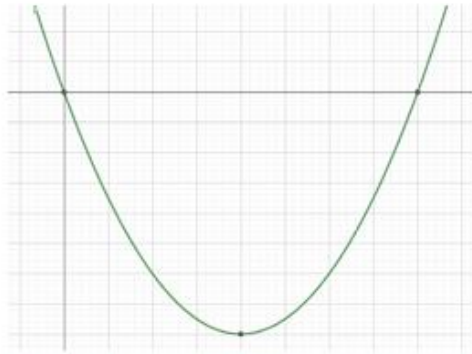
$$5x^2 - 40x = 5x(x - 8) \geq 0$$

M1



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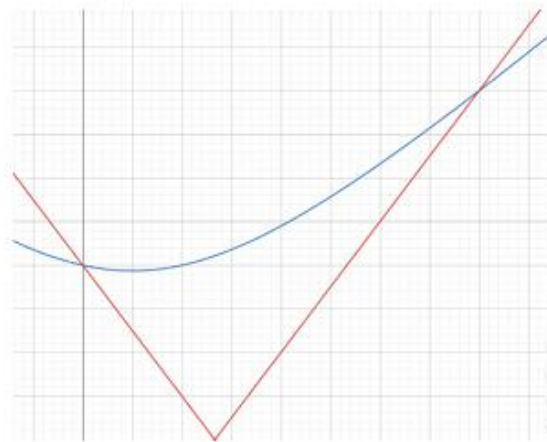
GRAPH G1 or consideration of intervals **M1**

[Alternative method

Solve for critical values

M1

Sketch graph



G1]

So $x \leq 0$ or $x \geq 8$

A1 (3)

(ii) (a)

$$\begin{aligned} & \left(\sqrt{4x^2 - 8x + 64} + 2(x - 1) \right) f(x) \\ &= \left(\sqrt{4x^2 - 8x + 64} + 2(x - 1) \right) \left(\sqrt{4x^2 - 8x + 64} - 2(x - 1) \right) \\ &= 4x^2 - 8x + 64 - 4x^2 + 8x - 4 = 60 \end{aligned}$$

Thus

$$f(x) = \frac{60}{\left(\sqrt{4x^2 - 8x + 64} + 2(x - 1) \right)}$$

and so $f(x) \rightarrow 0$ as $x \rightarrow \infty$

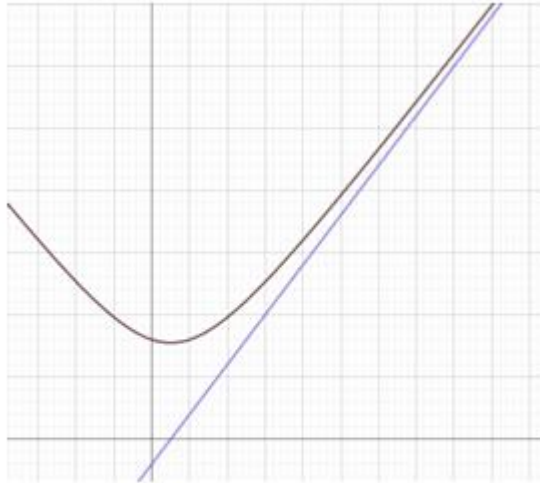
E1 (1)



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(b)



G2 (2)

(iii) Require one critical value 3, so $3m + c = \pm 5$

M1

and as only one critical value choose $m = 2$ and $c = -1$, or $m = -2$ and $c = 1$

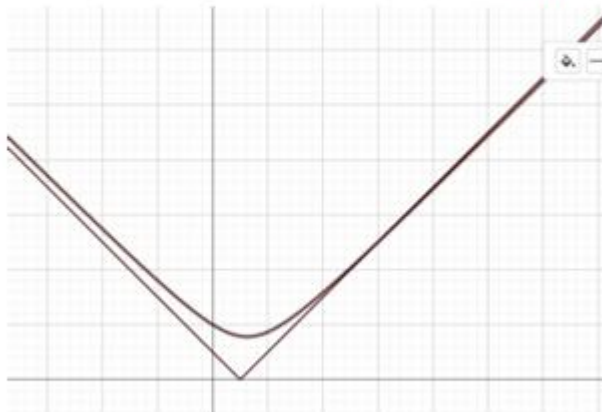
dM1 A1

$$\sqrt{4x^2 - 5x + 4} \leq |2x - 1|$$

$$4x^2 - 5x + 4 \leq (2x - 1)^2 = 4x^2 - 4x + 1$$

Giving $x \geq 3$

M1



G1 (5)



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(iv) To obtain 4 critical values require quadratic to cross x axis and so

E1

$$x^2 + px + q = mx + c$$

has roots -5 and 7 giving $p - m = -2$ and $q - c = -35$

M1 A1

and

$$-(x^2 + px + q) = mx + c$$

has roots 1 and 5 giving $p + m = -6$ and $q + c = 5$

B1

Thus, $p = -4$, $q = -15$, $m = -2$, $c = 20$

A1



G1 (6)



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