

## STEP III, 2022, Q1

- 1 Let  $C_1$  be the curve given by the parametric equations

$$x = ct, \quad y = \frac{c}{t},$$

where  $c > 0$  and  $t \neq 0$ , and let  $C_2$  be the circle

$$(x - a)^2 + (y - b)^2 = r^2.$$

$C_1$  and  $C_2$  intersect at the four points  $P_i$  ( $i = 1, 2, 3, 4$ ), and the corresponding values of the parameter  $t$  at these points are  $t_i$ .

- (i) Show that  $t_i$  are the roots of the equation

$$c^2 t^4 - 2act^3 + (a^2 + b^2 - r^2)t^2 - 2bct + c^2 = 0. \quad (*)$$

- (ii) Show that

$$\sum_{i=1}^4 t_i^2 = \frac{2}{c^2}(a^2 - b^2 + r^2)$$

and find a similar expression for  $\sum_{i=1}^4 \frac{1}{t_i^2}$ .

- (iii) Hence show that  $\sum_{i=1}^4 OP_i^2 = 4r^2$ , where  $OP_i$  denotes the distance of the point  $P_i$  from the origin.

- (iv) Suppose that the curves  $C_1$  and  $C_2$  touch at two distinct points.

By considering the product of the roots of (\*), or otherwise, show that the centre of circle  $C_2$  must lie on either the line  $y = x$  or  $y = -x$ .



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