

STEP III, 2021, Q3

- 3 (i) Let $I_n = \int_0^\beta (\sec x + \tan x)^n dx$, where n is a non-negative integer and $0 < \beta < \frac{\pi}{2}$.

For $n \geq 1$, show that

$$\frac{1}{2}(I_{n+1} + I_{n-1}) = \frac{1}{n}((\sec \beta + \tan \beta)^n - 1).$$

Show also that

$$I_n < \frac{1}{n}((\sec \beta + \tan \beta)^n - 1).$$

- (ii) Let $J_n = \int_0^\beta (\sec x \cos \beta + \tan x)^n dx$, where n is a non-negative integer and $0 < \beta < \frac{\pi}{2}$.

For $n \geq 1$, show that

$$J_n < \frac{1}{n}((1 + \tan \beta)^n - \cos^n \beta).$$



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