

## STEP III, 2020, Q8 EC

About 60% of the candidates attempted this question, but it was the second least successful question on the paper with a mean score of about one third marks. There were some very good solutions to this question, but most candidates only provided fragmentary answers, and stopped after the first couple of parts.

A common mistake was to use the condition  $u_{2k} = u_k$  along with  $u_1 = 1$  to erroneously conclude that all the terms with an even subscript are equal to 1. This might have been avoided if the candidates had written out the first 10 (or so) terms of the sequence to help them get a “feel” for what was happening, which could have also help stopped some other misconceptions along the way.

Part (i) was generally well done, but some candidates did not consider both cases  $u_{2k+1} > u_{2k}$  and  $u_{2k-1} > u_{2k}$ . Other candidates concluded that  $u_{k+1} + u_k > u_k$  without justifying this inequality by stating that all the terms are positive.

Part (ii) was attempted well by many candidates but was less successful than part (i). Some candidates who correctly considered both the  $(u_{2k-1}, u_{2k})$  and  $(u_{2k}, u_{2k+1})$  cases in part (i) then failed to consider both in this part. Some candidates erroneously assumed that if two terms  $p, q$  share a common factor and  $p < q$  then it must be the case that  $q = kp$ .

Part (iii) was only answered well by a few of the candidates. Some did not appreciate that “consecutively” means appears one directly after another, instead taking it to mean that the second one occurs at some position after the first one. Only a small minority of attempts considered both the  $(u_{2k-1}, u_{2k})$  and  $(u_{2k}, u_{2k+1})$  cases. A lot of candidates erroneously stated that “if  $u_k = a$  then if  $a$  is going to reappear then the next index must be  $2^n k$  for some integer  $n$ ”. A look at the first few terms of the sequence shows that  $u_5 = u_7 = 3$  which contradicts that statement.

Part (iv) was not well attempted. Some candidates did not process the wording (which was designed to help with the next part), and some tried to show instead that if  $a$  and  $b$  were two co-prime integers which **do** occur consecutively in the sequence etc.

The most successful candidates used contradiction here to show that if  $a - b$  and  $b$  do occur consecutively then this means that  $a$  and  $b$  must occur consecutively.

Some candidates correctly showed the first result, but when trying to find the similar result for  $a < b$  ended up with  $a$  following  $b$  and so essentially proved the same result again.

Part (v) was answered by only a few of the candidates attempting this question. There were some very well-reasoned arguments, including some candidates who used a construction method to justify that all possible rational numbers are in the range of  $f(n)$ . Only a very small number connected part (iv) to this part of the question.



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