

STEP III, 2020, Q7

- 7 (i) Given that the variables x , y and u are connected by the differential equations

$$\frac{du}{dx} + f(x)u = h(x) \quad \text{and} \quad \frac{dy}{dx} + g(x)y = u,$$

show that

$$\frac{d^2y}{dx^2} + (g(x) + f(x))\frac{dy}{dx} + (g'(x) + f(x)g(x))y = h(x). \quad (1)$$

- (ii) Given that the differential equation

$$\frac{d^2y}{dx^2} + \left(1 + \frac{4}{x}\right)\frac{dy}{dx} + \left(\frac{2}{x} + \frac{2}{x^2}\right)y = 4x + 12 \quad (2)$$

can be written in the same form as (1), find a first order differential equation which is satisfied by $g(x)$.

If $g(x) = kx^n$, find a possible value of n and the corresponding value of k .

Hence find a solution of (2) with $y = 5$ and $\frac{dy}{dx} = -3$ at $x = 1$.



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