

## STEP III, 2020, Q5

5 Show that for positive integer  $n$ ,  $x^n - y^n = (x - y) \sum_{r=1}^n x^{n-r} y^{r-1}$ .

(i) Let  $F$  be defined by

$$F(x) = \frac{1}{x^n(x - k)} \quad \text{for } x \neq 0, k$$

where  $n$  is a positive integer and  $k \neq 0$ .

(a) Given that

$$F(x) = \frac{A}{x - k} + \frac{f(x)}{x^n},$$

where  $A$  is a constant and  $f(x)$  is a polynomial, show that

$$f(x) = \frac{1}{x - k} \left( 1 - \left( \frac{x}{k} \right)^n \right).$$

Deduce that

$$F(x) = \frac{1}{k^n(x - k)} - \frac{1}{k} \sum_{r=1}^n \frac{1}{k^{n-r} x^r}.$$

(b) By differentiating  $x^n F(x)$ , prove that

$$\frac{1}{x^n(x - k)^2} = \frac{1}{k^n(x - k)^2} - \frac{n}{xk^n(x - k)} + \sum_{r=1}^n \frac{n - r}{k^{n+1-r} x^{r+1}}.$$

(ii) Hence evaluate the limit of

$$\int_2^N \frac{1}{x^3(x - 1)^2} dx$$

as  $N \rightarrow \infty$ , justifying your answer.



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