

STEP III, 2019, Q7 MS

7. (i)

$$y^2(y^2 - a^2) = x^2(x^2 - a^2)$$

$$y^4 - a^2y^2 + \frac{a^4}{4} = x^4 - a^2x^2 + \frac{a^4}{4}$$

M1

$$\left(y^2 - \frac{a^2}{2}\right)^2 = \left(x^2 - \frac{a^2}{2}\right)^2$$

$$y^2 - \frac{a^2}{2} = \pm \left(x^2 - \frac{a^2}{2}\right)$$

So $x^2 = y^2$ which gives $y = \pm x$

or $x^2 + y^2 = a^2$

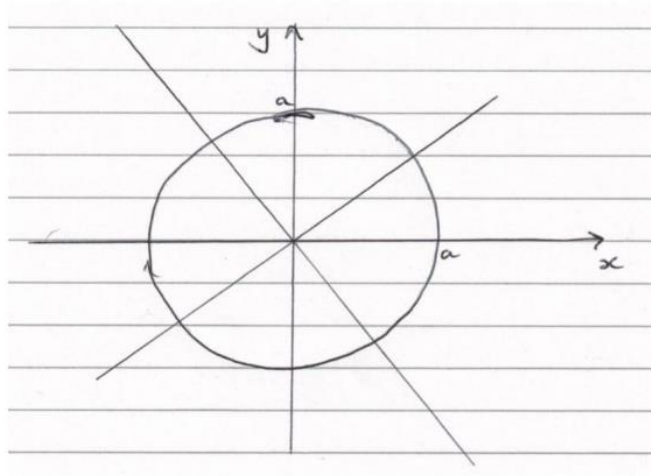
A1

Alternatively

$$y^4 - x^4 = a^2y^2 - a^2x^2$$

so $(y^2 - x^2)(y^2 + x^2) - a^2(y^2 - x^2) = 0$

M1



G1

[3]

(ii)

$$y^2(y^2 - 5) = x^2(x^2 - 4)$$

a)

$$(x^2)^2 - 4x^2 - y^2(y^2 - 5) = 0$$

So for real x^2 ,

$$16 + 4y^2(y^2 - 5) \geq 0$$



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M1

$$(y^2)^2 - 5y^2 + 4 \geq 0$$

$$(y^2 - 1)(y^2 - 4) \geq 0$$

$$(y - 1)(y + 1)(y - 2)(y + 2) \geq 0$$

M1

As $y \geq 0$,

$$(y - 1)(y - 2) \geq 0$$

So $0 \leq y \leq 1$ or $y \geq 2$

A1* [3]

b) For small x and y ,

$$y^4 - 5y^2 = x^4 - 4x^2 \text{ becomes } 5y^2 \approx 4x^2 \text{ so } y \approx \pm \frac{2x}{\sqrt{5}}$$

B1

For large x and y ,

$$y^4 - 5y^2 = x^4 - 4x^2 \text{ becomes } y^4 \approx x^4 \text{ so } y \approx \pm x$$

B1 [2]

c)

$$y^2(y^2 - 5) = x^2(x^2 - 4)$$

$$(4y^3 - 10y) \frac{dy}{dx} = 4x^3 - 8x$$

M1

$$\frac{dy}{dx} = 0 \Rightarrow 4x^3 - 8x = 0$$

$$4x(x^2 - 2) = 0$$

M1

So $x = 0, y = 0, \sqrt{5}$ or $x = \sqrt{2}, y = 1, 2$

Thus $(0, \sqrt{5}), (\sqrt{2}, 1), (\sqrt{2}, 2)$ but not $(0, 0)$

A1 B1

$$\frac{dx}{dy} = 0 \Rightarrow 4y^3 - 10y = 0$$

$$2y(2y^2 - 5) = 0$$

M1

So $y = 0, x = 0, 2$ but $y = \sqrt{\frac{5}{2}}$ gives x complex

E1

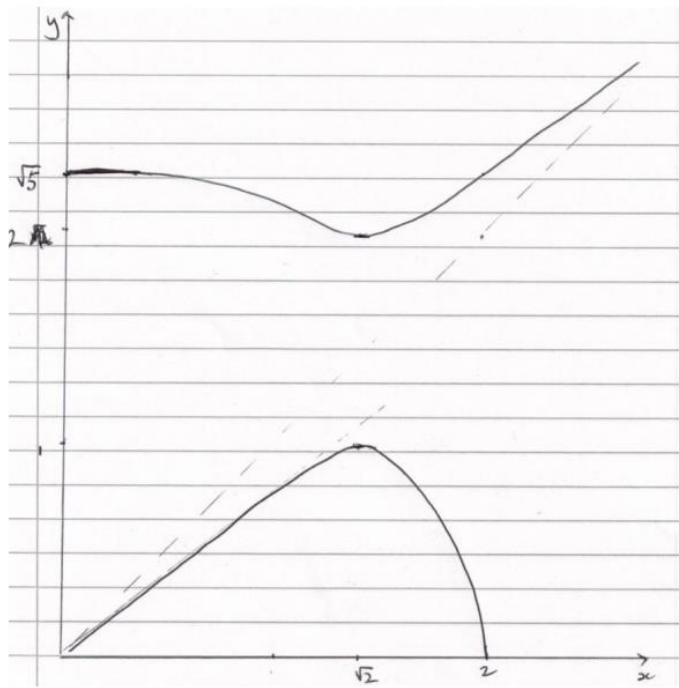
Thus $(2, 0)$ but not $(0, 0)$

A1 [7]



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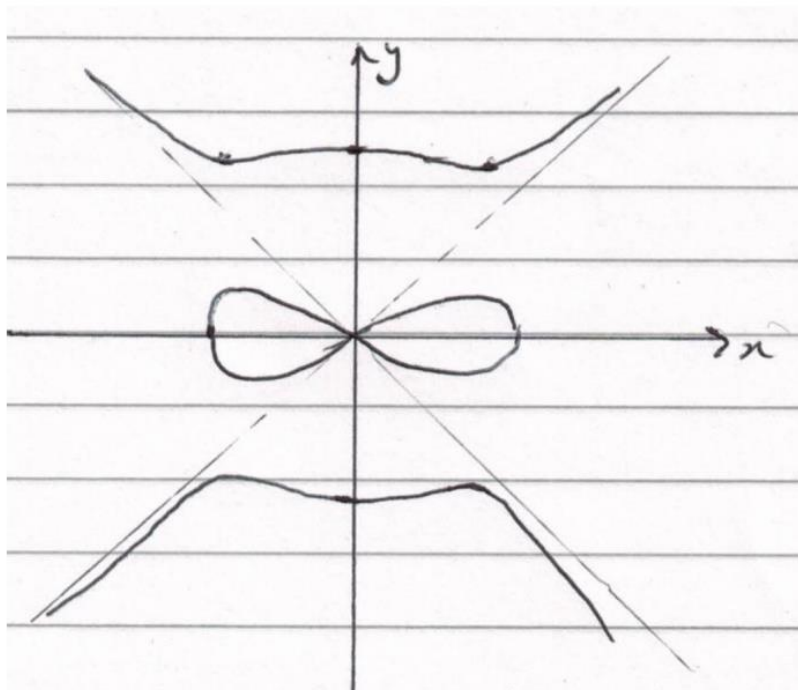


G1 G1 G1 G1

[4]

(iii) G1 ft

[1]



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