

## STEP III, 2019, Q7

7 The *Devil's Curve* is given by

$$y^2(y^2 - b^2) = x^2(x^2 - a^2),$$

where  $a$  and  $b$  are positive constants.

- (i) In the case  $a = b$ , sketch the Devil's Curve.
- (ii) Now consider the case  $a = 2$  and  $b = \sqrt{5}$ , and  $x \geq 0$ ,  $y \geq 0$ .
  - (a) Show by considering a quadratic equation in  $x^2$  that either  $0 \leq y \leq 1$  or  $y \geq 2$ .
  - (b) Describe the curve very close to and very far from the origin.
  - (c) Find the points at which the tangent to the curve is parallel to the  $x$ -axis and the point at which the tangent to the curve is parallel to the  $y$ -axis.

Sketch the Devil's Curve in this case.

- (iii) Sketch the Devil's Curve in the case  $a = 2$  and  $b = \sqrt{5}$  again, but with  $-\infty < x < \infty$  and  $-\infty < y < \infty$ .



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