

STEP III, 2019, Q2

- 2 The definition of the derivative f' of a (differentiable) function f is

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}. \quad (*)$$

- (i) The function f has derivative f' and satisfies

$$f(x+y) = f(x)f(y)$$

for all x and y , and $f'(0) = k$ where $k \neq 0$. Show that $f(0) = 1$.

Using (*), show that $f'(x) = kf(x)$ and find $f(x)$ in terms of x and k .

- (ii) The function g has derivative g' and satisfies

$$g(x+y) = \frac{g(x) + g(y)}{1 + g(x)g(y)}$$

for all x and y , $|g(x)| < 1$ for all x , and $g'(0) = k$ where $k \neq 0$.

Find $g'(x)$ in terms of $g(x)$ and k , and hence find $g(x)$ in terms of x and k .



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