

STEP III, 2018, Q3

- 3 Show that the second-order differential equation

$$x^2y'' + (1 - 2p)xy' + (p^2 - q^2)y = f(x),$$

where p and q are constants, can be written in the form

$$x^a(x^b(x^c y)')' = f(x), \quad (*)$$

where a , b and c are constants.

- (i) Use (*) to derive the general solution of the equation

$$x^2y'' + (1 - 2p)xy' + (p^2 - q^2)y = 0$$

in the different cases that arise according to the values of p and q .

- (ii) Use (*) to derive the general solution of the equation

$$x^2y'' + (1 - 2p)xy' + p^2y = x^n$$

in the different cases that arise according to the values of p and n .



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