

## STEP III, 2017, Q8

8 Prove that, for any numbers  $a_1, a_2, \dots$ , and  $b_1, b_2, \dots$ , and for  $n \geq 1$ ,

$$\sum_{m=1}^n a_m(b_{m+1} - b_m) = a_{n+1}b_{n+1} - a_1b_1 - \sum_{m=1}^n b_{m+1}(a_{m+1} - a_m).$$

(i) By setting  $b_m = \sin mx$ , show that

$$\sum_{m=1}^n \cos\left(m + \frac{1}{2}\right)x = \frac{1}{2}(\sin(n+1)x - \sin x) \operatorname{cosec} \frac{1}{2}x.$$

**Note:**  $\sin A - \sin B = 2 \cos\left(\frac{A+B}{2}\right) \sin\left(\frac{A-B}{2}\right).$

(ii) Show that

$$\sum_{m=1}^n m \sin mx = (p \sin(n+1)x + q \sin nx) \operatorname{cosec}^2 \frac{1}{2}x,$$

where  $p$  and  $q$  are to be determined in terms of  $n$ .

**Note:**  $2 \sin A \sin B = \cos(A-B) - \cos(A+B);$

$2 \cos A \sin B = \sin(A+B) - \sin(A-B).$



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