

## STEP III, 2017, Q5

- 5 The point with cartesian coordinates  $(x, y)$  lies on a curve with polar equation  $r = f(\theta)$ . Find an expression for  $\frac{dy}{dx}$  in terms of  $f(\theta)$ ,  $f'(\theta)$  and  $\tan \theta$ .

Two curves, with polar equations  $r = f(\theta)$  and  $r = g(\theta)$ , meet at right angles. Show that where they meet

$$f'(\theta)g'(\theta) + f(\theta)g(\theta) = 0.$$

The curve  $C$  has polar equation  $r = f(\theta)$  and passes through the point given by  $r = 4$ ,  $\theta = -\frac{1}{2}\pi$ . For each positive value of  $a$ , the curve with polar equation  $r = a(1 + \sin \theta)$  meets  $C$  at right angles. Find  $f(\theta)$ .

Sketch on a single diagram the three curves with polar equations  $r = 1 + \sin \theta$ ,  $r = 4(1 + \sin \theta)$  and  $r = f(\theta)$ .



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