STEP III, 2017, Q5

5 The point with cartesian coordinates (x, y) lies on a curve with polar equation $r = \mathrm{f}(\theta)$. Find an expression for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of $\mathrm{f}(\theta)$, $\mathrm{f}'(\theta)$ and $\tan\theta$.

Two curves, with polar equations $r = \mathrm{f}(\theta)$ and $r = \mathrm{g}(\theta)$, meet at right angles. Show that where they meet

$$\mathrm{f}'(\theta)\mathrm{g}'(\theta) + \mathrm{f}(\theta)\mathrm{g}(\theta) = 0\,.$$

The curve C has polar equation $r = \mathrm{f}(\theta)$ and passes through the point given by $r = 4$, $\theta = -\frac{1}{2}\pi$. For each positive value of a, the curve with polar equation $r = a(1 + \sin\theta)$ meets C at right angles. Find $\mathrm{f}(\theta)$.

Sketch on a single diagram the three curves with polar equations $r = 1+\sin\theta$, $r = 4(1+\sin\theta)$ and $r = \mathrm{f}(\theta)$.