

STEP III, 2017, Q3

- 3 Let α , β , γ and δ be the roots of the quartic equation

$$x^4 + px^3 + qx^2 + rx + s = 0.$$

You are given that, for any such equation, $\alpha\beta + \gamma\delta$, $\alpha\gamma + \beta\delta$ and $\alpha\delta + \beta\gamma$ satisfy a cubic equation of the form

$$y^3 + Ay^2 + (pr - 4s)y + (4qs - p^2s - r^2) = 0.$$

Determine A .

Now consider the quartic equation given by $p = 0$, $q = 3$, $r = -6$ and $s = 10$.

- (i) Find the value of $\alpha\beta + \gamma\delta$, given that it is the largest root of the corresponding cubic equation.
- (ii) Hence, using the values of q and s , find the value of $(\alpha + \beta)(\gamma + \delta)$ and the value of $\alpha\beta$ given that $\alpha\beta > \gamma\delta$.
- (iii) Using these results, and the values of p and r , solve the quartic equation.



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