

STEP III, 2017, Q2

- 2 The transformation R in the complex plane is a rotation (anticlockwise) by an angle θ about the point represented by the complex number a . The transformation S in the complex plane is a rotation (anticlockwise) by an angle ϕ about the point represented by the complex number b .

- (i) The point P is represented by the complex number z . Show that the image of P under R is represented by

$$e^{i\theta}z + a(1 - e^{i\theta}).$$

- (ii) Show that the transformation SR (equivalent to R followed by S) is a rotation about the point represented by c , where

$$c \sin \frac{1}{2}(\theta + \phi) = a e^{i\phi/2} \sin \frac{1}{2}\theta + b e^{-i\theta/2} \sin \frac{1}{2}\phi,$$

provided $\theta + \phi \neq 2n\pi$ for any integer n .

What is the transformation SR if $\theta + \phi = 2\pi$?

- (iii) Under what circumstances is $RS = SR$?



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