

STEP III, 2017, Q1

- 1 (i) Prove that, for any positive integers n and r ,

$$\frac{1}{{}^{n+r}C_{r+1}} = \frac{r+1}{r} \left(\frac{1}{{}^{n+r-1}C_r} - \frac{1}{{}^{n+r}C_r} \right).$$

Hence determine

$$\sum_{n=1}^{\infty} \frac{1}{{}^{n+r}C_{r+1}},$$

and deduce that $\sum_{n=2}^{\infty} \frac{1}{{}^{n+2}C_3} = \frac{1}{2}$.

- (ii) Show that, for $n \geq 3$,

$$\frac{3!}{n^3} < \frac{1}{{}^{n+1}C_3} \quad \text{and} \quad \frac{20}{{}^{n+1}C_3} - \frac{1}{{}^{n+2}C_5} < \frac{5!}{n^3}.$$

By summing these inequalities for $n \geq 3$, show that

$$\frac{115}{96} < \sum_{n=1}^{\infty} \frac{1}{n^3} < \frac{116}{96}.$$

Note: nC_r is another notation for $\binom{n}{r}$.



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to NextStepMaths.com