

STEP III, 2016 , Q8

- 8 (i) The function f satisfies, for all x , the equation

$$f(x) + (1 - x)f(-x) = x^2.$$

Show that $f(-x) + (1 + x)f(x) = x^2$. Hence find $f(x)$ in terms of x . You should verify that your function satisfies the original equation.

- (ii) The function K is defined, for $x \neq 1$, by

$$K(x) = \frac{x + 1}{x - 1}.$$

Show that, for $x \neq 1$, $K(K(x)) = x$.

The function g satisfies the equation

$$g(x) + xg\left(\frac{x + 1}{x - 1}\right) = x \quad (x \neq 1).$$

Show that, for $x \neq 1$, $g(x) = \frac{2x}{x^2 + 1}$.

- (iii) Find $h(x)$, for $x \neq 0$, $x \neq 1$, given that

$$h(x) + h\left(\frac{1}{1 - x}\right) = 1 - x - \frac{1}{1 - x} \quad (x \neq 0, \quad x \neq 1).$$



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