

## STEP III, 2016 , Q7

- 7 Let  $\omega = e^{2\pi i/n}$ , where  $n$  is a positive integer. Show that, for any complex number  $z$ ,

$$(z - 1)(z - \omega) \cdots (z - \omega^{n-1}) = z^n - 1.$$

The points  $X_0, X_1, \dots, X_{n-1}$  lie on a circle with centre  $O$  and radius 1, and are the vertices of a regular polygon.

- (i) The point  $P$  is equidistant from  $X_0$  and  $X_1$ . Show that, if  $n$  is even,

$$|PX_0| \times |PX_1| \times \cdots \times |PX_{n-1}| = |OP|^n + 1,$$

where  $|PX_k|$  denotes the distance from  $P$  to  $X_k$ .

Give the corresponding result when  $n$  is odd. (There are two cases to consider.)

- (ii) Show that

$$|X_0X_1| \times |X_0X_2| \times \cdots \times |X_0X_{n-1}| = n.$$



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