

STEP III, 2016 , Q6

- 6 Show, by finding R and γ , that $A \sinh x + B \cosh x$ can be written in the form $R \cosh(x + \gamma)$ if $B > A > 0$. Determine the corresponding forms in the other cases that arise, for $A > 0$, according to the value of B .

Two curves have equations $y = \operatorname{sech} x$ and $y = a \tanh x + b$, where $a > 0$.

- (i) In the case $b > a$, show that if the curves intersect then the x -coordinates of the points of intersection can be written in the form

$$\pm \operatorname{arcosh} \left(\frac{1}{\sqrt{b^2 - a^2}} \right) - \operatorname{artanh} \frac{a}{b}.$$

- (ii) Find the corresponding result in the case $a > b > 0$.
- (iii) Find necessary and sufficient conditions on a and b for the curves to intersect at two distinct points.
- (iv) Find necessary and sufficient conditions on a and b for the curves to touch and, given that they touch, express the y -coordinate of the point of contact in terms of a .



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