

STEP III, 2016 , Q4

- 4 (i) By considering $\frac{1}{1+x^r} - \frac{1}{1+x^{r+1}}$ for $|x| \neq 1$, simplify

$$\sum_{r=1}^N \frac{x^r}{(1+x^r)(1+x^{r+1})}.$$

Show that, for $|x| < 1$,

$$\sum_{r=1}^{\infty} \frac{x^r}{(1+x^r)(1+x^{r+1})} = \frac{x}{1-x^2}.$$

- (ii) Deduce that

$$\sum_{r=1}^{\infty} \operatorname{sech}(ry) \operatorname{sech}((r+1)y) = 2e^{-y} \operatorname{cosech}(2y)$$

for $y > 0$.

Hence simplify

$$\sum_{r=-\infty}^{\infty} \operatorname{sech}(ry) \operatorname{sech}((r+1)y),$$

for $y > 0$.



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