

STEP III, 2016 , Q2

- 2 The distinct points $P(ap^2, 2ap)$, $Q(aq^2, 2aq)$ and $R(ar^2, 2ar)$ lie on the parabola $y^2 = 4ax$, where $a > 0$. The points are such that the normal to the parabola at Q and the normal to the parabola at R both pass through P .
- (i) Show that $q^2 + qp + 2 = 0$.
- (ii) Show that QR passes through a certain point that is independent of the choice of P .
- (iii) Let T be the point of intersection of OP and QR , where O is the coordinate origin. Show that T lies on a line that is independent of the choice of P .
- Show further that the distance from the x -axis to T is less than $\frac{a}{\sqrt{2}}$.



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