

## STEP III, 2016 , Q13

- 13 Given a random variable  $X$  with mean  $\mu$  and standard deviation  $\sigma$ , we define the *kurtosis*,  $\kappa$ , of  $X$  by

$$\kappa = \frac{E((X - \mu)^4)}{\sigma^4} - 3.$$

Show that the random variable  $X - a$ , where  $a$  is a constant, has the same kurtosis as  $X$ .

- (i) Show by integration that a random variable which is Normally distributed with mean 0 has kurtosis 0.
- (ii) Let  $Y_1, Y_2, \dots, Y_n$  be  $n$  independent, identically distributed, random variables with mean 0, and let  $T = \sum_{r=1}^n Y_r$ . Show that

$$E(T^4) = \sum_{r=1}^n E(Y_r^4) + 6 \sum_{r=1}^{n-1} \sum_{s=r+1}^n E(Y_s^2)E(Y_r^2).$$

- (iii) Let  $X_1, X_2, \dots, X_n$  be  $n$  independent, identically distributed, random variables each with kurtosis  $\kappa$ . Show that the kurtosis of their sum is  $\frac{\kappa}{n}$ .



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