STEP III, 2016 , Q12 MS

12. Using the binomial distribution, $\mu = 20n$, $\sigma^2 = 16n$, writing $16n \le X \le 24n$ as $|X - 20n| \le 4n$ enables Chebyshev to be applied with $k = \sqrt{n}$ leading to the required result in (i). Similarly, in part (ii), considering a Poisson distribution with mean n , and appreciating that $|X - n| > n$ implies $X > 2n$ in these circumstances, the same value of k as in part (i) with Chebyshev leads to the desired result.

12. (i) $X \sim B(100n, 0.2)$ **B1**

So $\mu = 100n \times 0.2 = 20n$ **M1 A1** and $\sigma^2 = 100n \times 0.2 \times 0.8 = 16n$ **M1 A1**

So $P(16n \le X \le 24n) = P(|X - 20n| \le 4n) = P\left(|X - 20n| \le \sqrt{n} \times \sqrt{16n}\right)$

M1 **M1 A1**

So by Chebyshev, $P(16n \le X \le 24n) \ge 1 - \left(\frac{1}{\sqrt{n}}\right)^2 = 1 - \frac{1}{n}$ as required. **A1* (9)**

(ii) Suppose $X \sim Po(n)$ **B1**

Then $\mu = n$ **B1** and $\sigma^2 = n$ **B1**

By Chebyshev, $P(|X - \mu| > k\sigma) \le \frac{1}{k^2}$

so let $k = \sqrt{n}$ and hence $P(|X - n| > n) \le \frac{1}{n}$

M1 **A1**

$$P(|X - n| > n) = P(X < 0 \; or X > 2n) = P(X > 2n) = 1 - e^{-n} - ne^{-n} - \frac{n^2 e^{-n}}{2!} - \cdots - \frac{n^{2n} e^{-n}}{2n!}$$

M1 **A1** **A1**

So $1 - e^{-n}\left(1 + n + \frac{n^2}{2!} + \cdots + \frac{n^{2n}}{2n!}\right) \le \frac{1}{n}$ **M1 A1**

and hence $1 + n + \frac{n^2}{2!} + \cdots + \frac{n^{2n}}{2n!} \ge \left(1 - \frac{1}{n}\right) e^n$ **A1* (11)**