

STEP III, 2016 , Q12

- 12 Let X be a random variable with mean μ and standard deviation σ . *Chebyshev's inequality*, which you may use without proof, is

$$P(|X - \mu| > k\sigma) \leq \frac{1}{k^2},$$

where k is any positive number.

- (i) The probability of a biased coin landing heads up is 0.2. It is thrown $100n$ times, where n is an integer greater than 1. Let α be the probability that the coin lands heads up N times, where $16n \leq N \leq 24n$.

Use Chebyshev's inequality to show that

$$\alpha \geq 1 - \frac{1}{n}.$$

- (ii) Use Chebyshev's inequality to show that

$$1 + n + \frac{n^2}{2!} + \cdots + \frac{n^{2n}}{(2n)!} \geq \left(1 - \frac{1}{n}\right) e^n.$$



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