Attempt this question independently, showing full and clear working.

STEP III 2016 Question 1

Let

$$I_n = \int_{-\infty}^{\infty} \frac{1}{(x^2+2ax+b)^n}\,\mathrm{d}x,$$

where a and b are constants with $b > a^2$, and n is a positive integer.

(i) By using the substitution $x + a = \sqrt{b-a^2}\tan u$, or otherwise, show that

$$I_1 = \frac{\pi}{\sqrt{b-a^2}}.$$

(ii) Show that $2n(b-a^2)\,I_{n+1} = (2n-1)\,I_n$.

(iii) Hence prove by induction that

$$I_n = \frac{\pi}{2^{2n-2}(b-a^2)^{n-\frac{1}{2}}}\binom{2n-2}{n-1}.$$