

STEP III, 2015 , Q8

- 8 (i) Show that under the changes of variable $x = r \cos \theta$ and $y = r \sin \theta$, where r is a function of θ with $r > 0$, the differential equation

$$(y + x) \frac{dy}{dx} = y - x$$

becomes

$$\frac{dr}{d\theta} + r = 0.$$

Sketch a solution in the x - y plane.

- (ii) Show that the solutions of

$$(y + x - x(x^2 + y^2)) \frac{dy}{dx} = y - x - y(x^2 + y^2)$$

can be written in the form

$$r^2 = \frac{1}{1 + Ae^{2\theta}}$$

and sketch the different forms of solution that arise according to the value of A .



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