

STEP III, 2015 , Q7

- 7 An operator D is defined, for any function f , by

$$Df(x) = x \frac{df(x)}{dx}.$$

The notation D^n means that D is applied n times; for example

$$D^2f(x) = x \frac{d}{dx} \left(x \frac{df(x)}{dx} \right).$$

Show that, for any constant a , $D^2x^a = a^2x^a$.

- (i) Show that if $P(x)$ is a polynomial of degree r (where $r \geq 1$) then, for any positive integer n , $D^n P(x)$ is also a polynomial of degree r .
- (ii) Show that if n and m are positive integers with $n < m$, then $D^n(1-x)^m$ is divisible by $(1-x)^{m-n}$.
- (iii) Deduce that, if m and n are positive integers with $n < m$, then

$$\sum_{r=0}^m (-1)^r \binom{m}{r} r^n = 0.$$



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