

## STEP III, 2015 , Q6

- 6 (i) Let  $w$  and  $z$  be complex numbers, and let  $u = w + z$  and  $v = w^2 + z^2$ . Prove that  $w$  and  $z$  are real if and only if  $u$  and  $v$  are real and  $u^2 \leq 2v$ .
- (ii) The complex numbers  $u$ ,  $w$  and  $z$  satisfy the equations

$$\begin{aligned}w + z - u &= 0 \\w^2 + z^2 - u^2 &= -\frac{2}{3} \\w^3 + z^3 - \lambda u &= -\lambda\end{aligned}$$

where  $\lambda$  is a positive real number. Show that for all values of  $\lambda$  except one (which you should find) there are three possible values of  $u$ , all real.

Are  $w$  and  $z$  necessarily real? Give a proof or counterexample.



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