

## STEP III, 2015 , Q5

- 5 (i) In the following argument to show that  $\sqrt{2}$  is irrational, give proofs appropriate for steps 3, 5 and 6.
1. Assume that  $\sqrt{2}$  is rational.
  2. Define the set  $S$  to be the set of positive integers with the following property:  
$$n \text{ is in } S \text{ if and only if } n\sqrt{2} \text{ is an integer.}$$
  3. Show that the set  $S$  contains at least one positive integer.
  4. Define the integer  $k$  to be the smallest positive integer in  $S$ .
  5. Show that  $(\sqrt{2} - 1)k$  is in  $S$ .
  6. Show that steps 4 and 5 are contradictory and hence that  $\sqrt{2}$  is irrational.
- (ii) Prove that  $2^{\frac{1}{3}}$  is rational if and only if  $2^{\frac{2}{3}}$  is rational.  
Use an argument similar to that of part (i) to prove that  $2^{\frac{1}{3}}$  and  $2^{\frac{2}{3}}$  are irrational.



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