

STEP III, 2015 , Q4

- 4 (i) If a , b and c are all real, show that the equation

$$z^3 + az^2 + bz + c = 0 \quad (*)$$

has at least one real root.

- (ii) Let

$$S_1 = z_1 + z_2 + z_3, \quad S_2 = z_1^2 + z_2^2 + z_3^2, \quad S_3 = z_1^3 + z_2^3 + z_3^3,$$

where z_1 , z_2 and z_3 are the roots of the equation (*). Express a and b in terms of S_1 and S_2 , and show that

$$6c = -S_1^3 + 3S_1S_2 - 2S_3.$$

- (iii) The six real numbers r_k and θ_k ($k = 1, 2, 3$), where $r_k > 0$ and $-\pi < \theta_k < \pi$, satisfy

$$\sum_{k=1}^3 r_k \sin(\theta_k) = 0, \quad \sum_{k=1}^3 r_k^2 \sin(2\theta_k) = 0, \quad \sum_{k=1}^3 r_k^3 \sin(3\theta_k) = 0.$$

Show that $\theta_k = 0$ for at least one value of k .

Show further that if $\theta_1 = 0$ then $\theta_2 = -\theta_3$.



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