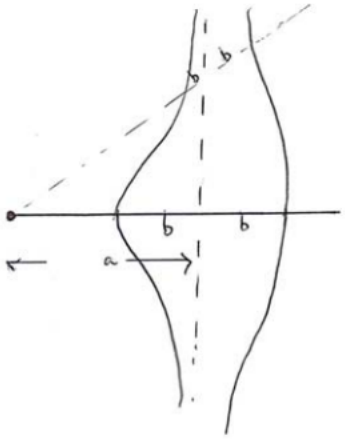
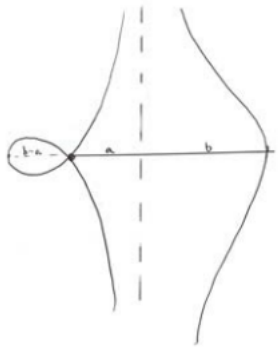
STEP III, 2015 , Q3 MS

3. The part (i) inequality for $\sec\theta$ can be obtained by making r the subject of the formula as $r = a\sec\theta \pm b$ and invoking $a > b$ remembering that $r < 0$ is not permitted.

Then the points lie on a conchoid of Nicomedes with *A* being the pole (origin), *d* being *b*, and L being the line $r = a\sec\theta$ ("x" $= a$). A sketch is

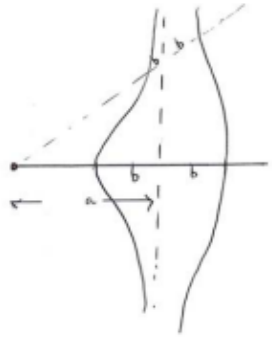


In part (ii), the extra feature is the loop as specified with end-points at the pole corresponding to $\sec\theta = \frac{-b}{a}$. A sketch is



So in the given case, the area is given by $2\times\frac{1}{2}\int_{\frac{2\pi}{3}}^{\pi}(\sec\theta+2)^2 d\theta$ which is $\frac{4\pi}{3}+\sqrt{3}-4\ln\left|2+\sqrt{3}\right|$.

3. (i)



Symmetry about initial line G1
Two branches G1
Shape and labelling G1 (3)

If $|r - a\sec\theta| = b$, then $r - a\sec\theta = b$ or $r - a\sec\theta = -b$

So $r = a\sec\theta + b$ or $r = a\sec\theta - b$ M1A1

If $\sec\theta < 0$, $a\sec\theta + b < -a + b < 0$ as $a > b$ and $a\sec\theta - b < -a - b < 0$ as a and b are both positive, and thus in both cases, $r < 0$ which is not permitted. B1

If $\sec\theta > 0$, $a\sec\theta + b > a + b > 0$ and $a\sec\theta - b > a - b > 0$ giving $r > 0$

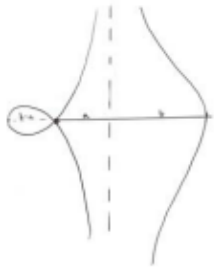
so $\sec\theta > 0$ as required. B1 (4)

So $r = a\sec\theta \pm b$, thus points satisfying (*) lie on a certain conchoid of Nicomedes with *A* being the pole (origin), B1

d being *b*, B1

and L being the line $r = a\sec\theta$. B1 (3)

(ii)



Symmetry about initial line G1
Two branches G1
Loop, shape and labelling G1

If $a < b$, then the curve has two branches, $r = a\sec\theta + b$ with $\sec\theta > 0$ and $r = a\sec\theta + b$ with $\sec\theta < 0$, the endpoints of the loop corresponding to $\sec\theta = \frac{-b}{a}$. **B1 (4)**

In the case $a = 1$ and $b = 2$, $\sec\theta = \frac{-2}{1} = -2$ so $\theta = \pm\frac{2\pi}{3}$

Area of loop

$= 2 \times \frac{1}{2}\int_{\frac{2\pi}{3}}^{\pi}(\sec\theta + 2)^2 d\theta$ **M1A1**

$= \int_{\frac{2\pi}{3}}^{\pi}\sec^2\theta + 4\sec\theta + 4\, d\theta = [\tan\theta + 4\ln|\sec\theta + \tan\theta| + 4\theta]_{\frac{2\pi}{3}}^{\pi}$ **M1A1**

$= 4\pi - \left(-\sqrt{3} + 4\ln|-2-\sqrt{3}| + \frac{8\pi}{3}\right) = \frac{4\pi}{3} + \sqrt{3} - 4\ln|2+\sqrt{3}|$ **M1A1 (6)**