

STEP III, 2015 , Q3

- 3 In this question, r and θ are polar coordinates with $r \geq 0$ and $-\pi < \theta \leq \pi$, and a and b are positive constants.

Let L be a fixed line and let A be a fixed point not lying on L . Then the locus of points that are a fixed distance (call it d) from L measured along lines through A is called a *conchoid of Nicomedes*.

- (i) Show that if

$$|r - a \sec \theta| = b, \quad (*)$$

where $a > b$, then $\sec \theta > 0$. Show that all points with coordinates satisfying (*) lie on a certain conchoid of Nicomedes (you should identify L , d and A). Sketch the locus of these points.

- (ii) In the case $a < b$, sketch the curve (including the loop for which $\sec \theta < 0$) given by

$$|r - a \sec \theta| = b.$$

Find the area of the loop in the case $a = 1$ and $b = 2$.

[Note: $\int \sec \theta \, d\theta = \ln |\sec \theta + \tan \theta| + C$.]



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