

## STEP III, 2015 , Q13

13 Each of the two independent random variables  $X$  and  $Y$  is uniformly distributed on the interval  $[0, 1]$ .

- (i) By considering the lines  $x + y = \text{constant}$  in the  $x$ - $y$  plane, find the cumulative distribution function of  $X + Y$ .

Hence show that the probability density function  $f$  of  $(X + Y)^{-1}$  is given by

$$f(t) = \begin{cases} 2t^{-2} - t^{-3} & \text{for } \frac{1}{2} \leq t \leq 1 \\ t^{-3} & \text{for } 1 \leq t < \infty \\ 0 & \text{otherwise.} \end{cases}$$

Evaluate  $E\left(\frac{1}{X + Y}\right)$ .

- (ii) Find the cumulative distribution function of  $Y/X$  and use this result to find the probability density function of  $\frac{X}{X + Y}$ .

Write down  $E\left(\frac{X}{X + Y}\right)$  and verify your result by integration.



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