

STEP III, 2015 , Q12

- 12 A 6-sided fair die has the numbers 1, 2, 3, 4, 5, 6 on its faces. The die is thrown n times, the outcome (the number on the top face) of each throw being independent of the outcome of any other throw. The random variable S_n is the sum of the outcomes.
- (i) The random variable R_n is the remainder when S_n is divided by 6. Write down the probability generating function, $G(x)$, of R_1 and show that the probability generating function of R_2 is also $G(x)$. Use a generating function to find the probability that S_n is divisible by 6.
- (ii) The random variable T_n is the remainder when S_n is divided by 5. Write down the probability generating function, $G_1(x)$, of T_1 and show that $G_2(x)$, the probability generating function of T_2 , is given by

$$G_2(x) = \frac{1}{36}(x^2 + 7y)$$

where $y = 1 + x + x^2 + x^3 + x^4$.

Obtain the probability generating function of T_n and hence show that the probability that S_n is divisible by 5 is

$$\frac{1}{5} \left(1 - \frac{1}{6^n} \right)$$

if n is not divisible by 5. What is the corresponding probability if n is divisible by 5?



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