

## STEP III, 2014, Q9

- 9 A particle of mass  $m$  is projected with velocity  $\mathbf{u}$ . It is acted upon by the force  $m\mathbf{g}$  due to gravity and by a resistive force  $-mk\mathbf{v}$ , where  $\mathbf{v}$  is its velocity and  $k$  is a positive constant. Given that, at time  $t$  after projection, its position  $\mathbf{r}$  relative to the point of projection is given by

$$\mathbf{r} = \frac{kt - 1 + e^{-kt}}{k^2} \mathbf{g} + \frac{1 - e^{-kt}}{k} \mathbf{u},$$

find an expression for  $\mathbf{v}$  in terms of  $k$ ,  $t$ ,  $\mathbf{g}$  and  $\mathbf{u}$ . Verify that the equation of motion and the initial conditions are satisfied.

Let  $\mathbf{u} = u \cos \alpha \mathbf{i} + u \sin \alpha \mathbf{j}$  and  $\mathbf{g} = -g\mathbf{j}$ , where  $0 < \alpha < 90^\circ$ , and let  $T$  be the time after projection at which  $\mathbf{r} \cdot \mathbf{j} = 0$ . Show that

$$uk \sin \alpha = \left( \frac{kT}{1 - e^{-kT}} - 1 \right) g.$$

Let  $\beta$  be the acute angle between  $\mathbf{v}$  and  $\mathbf{i}$  at time  $T$ . Show that

$$\tan \beta = \frac{(e^{kT} - 1)g}{uk \cos \alpha} - \tan \alpha.$$

Show further that  $\tan \beta > \tan \alpha$  (you may assume that  $\sinh kT > kT$ ) and deduce that  $\beta > \alpha$ .



# NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to [NextStepMaths.com](http://NextStepMaths.com)