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8. The initial result is obtained by extending the given inequality so that each term of the sum is compared with $f(k^n)$ and $f(k^{n+1})$. Part (i) is obtained using the stem, the given function, $= 2$, and summing the sums. The deduction relies on considering the lower limit of the sum. The same approach applies to part (ii), with the new function given and considering the upper limit which is obtained as a geometric progression. Counting the number of elements of $S(1000)$ gives the method for obtaining $\sigma(n)$ using the same function as part (i) except $f(r) = 0$ if r has one or more 2 s in its decimal representation and with $k = 10$, again with the sum of a geometric progression. The final result is particularly attractive, demonstrating how few terms need to be removed from the non-convergent harmonic progression (of part (i)) in order to produce a convergent sequence.