

STEP III, 2014, Q8

- 8 The numbers $f(r)$ satisfy $f(r) > f(r + 1)$ for $r = 1, 2, \dots$. Show that, for any non-negative integer n ,

$$k^n(k-1)f(k^{n+1}) \leq \sum_{r=k^n}^{k^{n+1}-1} f(r) \leq k^n(k-1)f(k^n)$$

where k is an integer greater than 1.

- (i) By taking $f(r) = 1/r$, show that

$$\frac{N+1}{2} \leq \sum_{r=1}^{2^{N+1}-1} \frac{1}{r} \leq N+1.$$

Deduce that the sum $\sum_{r=1}^{\infty} \frac{1}{r}$ does not converge.

- (ii) By taking $f(r) = 1/r^3$, show that

$$\sum_{r=1}^{\infty} \frac{1}{r^3} \leq 1\frac{1}{3}.$$

- (iii) Let $S(n)$ be the set of positive integers less than n which do not have a 2 in their decimal representation and let $\sigma(n)$ be the sum of the reciprocals of the numbers in $S(n)$, so for example $\sigma(5) = 1 + \frac{1}{3} + \frac{1}{4}$. Show that $S(1000)$ contains $9^3 - 1$ distinct numbers.

Show that $\sigma(n) < 80$ for all n .



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