

STEP III, 2014 , Q7

7 The four distinct points P_i ($i = 1, 2, 3, 4$) are the vertices, labelled anticlockwise, of a cyclic quadrilateral. The lines P_1P_3 and P_2P_4 intersect at Q .

(i) By considering the triangles P_1QP_4 and P_2QP_3 show that $(P_1Q)(QP_3) = (P_2Q)(QP_4)$.

(ii) Let \mathbf{p}_i be the position vector of the point P_i ($i = 1, 2, 3, 4$). Show that there exist numbers a_i , not all zero, such that

$$\sum_{i=1}^4 a_i = 0 \quad \text{and} \quad \sum_{i=1}^4 a_i \mathbf{p}_i = \mathbf{0}. \quad (*)$$

(iii) Let a_i ($i = 1, 2, 3, 4$) be any numbers, not all zero, that satisfy (*). Show that $a_1 + a_3 \neq 0$ and that the lines P_1P_3 and P_2P_4 intersect at the point with position vector

$$\frac{a_1 \mathbf{p}_1 + a_3 \mathbf{p}_3}{a_1 + a_3}.$$

Deduce that $a_1 a_3 (P_1P_3)^2 = a_2 a_4 (P_2P_4)^2$.



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