STEP III, 2014 , Q5 MS

5. ABCD is a parallelogram if and only if $\overrightarrow{AB} = \overrightarrow{DC}$ which yields the required result. To be a square as well, angle $ABC = 90^o$, and $|AB| = |BC|$, so $c - b = i(b - a)$. Treating the two results as simultaneous equations to be solved for a and c in terms of b and d, the second result of the stem can be shown with reversible logic. For part (i) the same logic can be used for PXQ as just used for ABC. From the stem, XYZT is a square if and only if $i(x - z) = y - t$, and

$x + z = y + t$ and given the working for X in part (i), these can be shown to be true treating Y, Z, and T similarly