

STEP III, 2014 , Q5

- 5 A quadrilateral drawn in the complex plane has vertices A, B, C and D , labelled anticlockwise. These vertices are represented, respectively, by the complex numbers a, b, c and d . Show that $ABCD$ is a parallelogram (defined as a quadrilateral in which opposite sides are parallel and equal in length) if and only if $a + c = b + d$. Show further that, in this case, $ABCD$ is a square if and only if $i(a - c) = b - d$.

Let $PQRS$ be a quadrilateral in the complex plane, with vertices labelled anticlockwise, the internal angles of which are all less than 180° . Squares with centres X, Y, Z and T are constructed externally to the quadrilateral on the sides PQ, QR, RS and SP , respectively.

- (i) If P and Q are represented by the complex numbers p and q , respectively, show that X can be represented by

$$\frac{1}{2}(p(1+i) + q(1-i)).$$

- (ii) Show that $XYZT$ is a square if and only if $PQRS$ is a parallelogram.



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