STEP III, 2014 , Q4 MS

4. Expanding the bracket in the integral I_1 , and employing $\sec^2 x = 1 + \tan^2 x$ yields I plus the integral of a perfect differential which can be evaluated simply. For $I = 0$, $y' + y \tan x = 0$ over the interval which can be solved using an integrating factor and then the condition $y = 0 , x = 1$ enables the arbitrary constant to be evaluated giving the required result. In part (ii), similar working can be undertaken with the integral which is to be considered, given $b = a$. The argument requires no discontinuity in the interval so $a < \frac{\pi}{2}$. The function $y = \cos ax$ can be shown to meet the requirement.