

STEP III, 2014, Q4

4 (i) Let

$$I = \int_0^1 ((y')^2 - y^2) dx \quad \text{and} \quad I_1 = \int_0^1 (y' + y \tan x)^2 dx,$$

where y is a given function of x satisfying $y = 0$ at $x = 1$. Show that $I - I_1 = 0$ and deduce that $I \geq 0$. Show further that $I = 0$ only if $y = 0$ for all x ($0 \leq x \leq 1$).

(ii) Let

$$J = \int_0^1 ((y')^2 - a^2 y^2) dx,$$

where a is a given positive constant and y is a given function of x , not identically zero, satisfying $y = 0$ at $x = 1$. By considering an integral of the form

$$\int_0^1 (y' + ay \tan bx)^2 dx,$$

where b is suitably chosen, show that $J \geq 0$. You should state the range of values of a , in the form $a < k$, for which your proof is valid.

In the case $a = k$, find a function y (not everywhere zero) such that $J = 0$.



NextStepMaths.com

To view mark schemes, fully worked solutions and examiner's comments, and for more details about tutoring and other services offered, go to NextStepMaths.com