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3. (i) Given that the shortest distance between the line and the parabola will be zero if they meet, investigating the solution of the equations simultaneously , and the discriminant of the resulting quadratic equation, the first result of the question is the case that they do not meet. The closest approach is the perpendicular distance of the point on the parabola where the tangent is parallel to the line, so using the standard parametric form, it is the perpendicular distance of $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$ from $= mx + c$, giving the required result with care being taken over the sign of the numerator bearing in mind the inequalities.

(ii) The shortest distance of a point on the axis from the parabola, is either the distance from the vertex to the point, or the distance along one of the normals (which are symmetrically situated) which is not the axis. If the normal at $(at^2, 2at)$ passes through $(p, 0)$, then $= 2a + at^2$. From this it can be simply shown that shortest distance is p if $\frac{p}{a} < 2$, and is $2\sqrt{a(p-a)}$ if $\frac{p}{a} \geq 2$.

Then for the circle, the results follow simply, that the shortest distance will be $p - b$ if $p > b$, and 0 otherwise if $\frac{p}{a} < 2$, and $2\sqrt{a(p-a)} - b$ if $4a(p-a) > b^2$ or 0 otherwise if $\frac{p}{a} \geq 2$.