

## STEP III, 2014 , Q13

**13** I play a game which has repeated rounds. Before the first round, my score is 0. Each round can have three outcomes:

1. my score is unchanged and the game ends;
2. my score is unchanged and I continue to the next round;
3. my score is increased by one and I continue to the next round.

The probabilities of these outcomes are  $a$ ,  $b$  and  $c$ , respectively (the same in each round), where  $a + b + c = 1$  and  $abc \neq 0$ . The random variable  $N$  represents my score at the end of a randomly chosen game.

Let  $G(t)$  be the probability generating function of  $N$ .

- (i) Suppose in the first round, the game ends. Show that the probability generating function conditional on this happening is 1.
- (ii) Suppose in the first round, the game continues to the next round with no change in score. Show that the probability generating function conditional on this happening is  $G(t)$ .
- (iii) By comparing the coefficients of  $t^n$ , show that  $G(t) = a + bG(t) + ctG(t)$ . Deduce that, for  $n \geq 0$ ,

$$P(N = n) = \frac{ac^n}{(1 - b)^{n+1}}.$$

- (iv) Show further that, for  $n \geq 0$ ,

$$P(N = n) = \frac{\mu^n}{(1 + \mu)^{n+1}},$$

where  $\mu = E(N)$ .



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