

STEP III, 2014 , Q12

12 The random variable X has probability density function $f(x)$ (which you may assume is differentiable) and cumulative distribution function $F(x)$ where $-\infty < x < \infty$. The random variable Y is defined by $Y = e^X$. You may assume throughout this question that X and Y have unique modes.

- (i) Find the median value y_m of Y in terms of the median value x_m of X .
- (ii) Show that the probability density function of Y is $f(\ln y)/y$, and deduce that the mode λ of Y satisfies $f'(\ln \lambda) = f(\ln \lambda)$.
- (iii) Suppose now that $X \sim N(\mu, \sigma^2)$, so that

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/(2\sigma^2)}.$$

Explain why

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-(x-\mu-\sigma^2)^2/(2\sigma^2)} dx = 1$$

and hence show that $E(Y) = e^{\mu + \frac{1}{2}\sigma^2}$.

- (iv) Show that, when $X \sim N(\mu, \sigma^2)$,

$$\lambda < y_m < E(Y).$$



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