

STEP III, 2014, Q1

- 1 Let a , b and c be real numbers such that $a + b + c = 0$ and let

$$(1 + ax)(1 + bx)(1 + cx) = 1 + qx^2 + rx^3$$

for all real x . Show that $q = bc + ca + ab$ and $r = abc$.

- (i) Show that the coefficient of x^n in the series expansion (in ascending powers of x) of $\ln(1 + qx^2 + rx^3)$ is $(-1)^{n+1}S_n$ where

$$S_n = \frac{a^n + b^n + c^n}{n}, \quad (n \geq 1).$$

- (ii) Find, in terms of q and r , the coefficients of x^2 , x^3 and x^5 in the series expansion (in ascending powers of x) of $\ln(1 + qx^2 + rx^3)$ and hence show that $S_2S_3 = S_5$.

- (iii) Show that $S_2S_5 = S_7$.

- (iv) Give a proof of, or find a counterexample to, the claim that $S_2S_7 = S_9$.



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