

STEP III, 2013 , Q9

- 9 A sphere of radius R and uniform density ρ_s is floating in a large tank of liquid of uniform density ρ . Given that the centre of the sphere is a distance x above the level of the liquid, where $x < R$, show that the volume of liquid displaced is

$$\frac{\pi}{3}(2R^3 - 3R^2x + x^3).$$

The sphere is acted upon by two forces only: its weight and an upward force equal in magnitude to the weight of the liquid it has displaced. Show that

$$4R^3\rho_s(g + \ddot{x}) = (2R^3 - 3R^2x + x^3)\rho g.$$

Given that the sphere is in equilibrium when $x = \frac{1}{2}R$, find ρ_s in terms of ρ . Find, in terms of R and g , the period of small oscillations about this equilibrium position.



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